Membrane Curvature Theory

A Candidate Mathematical Framework

From galaxy rotation curves to a five-dimensional brane, lensing, cosmology, structure growth, directional expansion and boundary optics

Purpose. This document develops one explicit mathematical research programme inspired by the conceptual proposal at darkmatterenergytheory.com. It combines the earlier nonlinear galaxy equation with the remaining equations needed to discuss relativistic gravity, gravitational lensing, cosmic expansion, perturbations, anisotropy and light crossing a boundary between neighbouring regions.

Status. The website does not uniquely determine these equations. Some results below are established brane-world or modified-gravity mathematics; some are consequences of a stated ansatz; others are deliberately marked as optional closures. The combined framework is not a validated theory and should not be presented as one until a single action, a well-posed initial-value problem and joint fits to data have been completed.

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Abstract

The conceptual theory proposes four linked effects: extended curvature around baryonic matter, accelerated expansion from membrane tension, direction-dependent expansion, and photons entering our observable region from neighbouring universe-regions. The first part of this document formulates extended curvature using a nonlinear AQUAL-type Poisson equation and derives its spherical solution, baryonic Tully-Fisher relation, effective curvature density, external-field response and lensing signal. The second part embeds the discussion in a five-dimensional brane framework using the bulk action, Israel junction condition and projected Einstein equation. Because the projected equations are not closed, a covariant preferred-frame strain sector is supplied as one candidate closure whose static limit reproduces the chosen galaxy equation. The remaining parts derive a brane Friedmann equation, a consistent dynamical strain field, linear-growth and lensing parameterisations, Bianchi-I directional expansion, Maxwell and junction conditions at a boundary, and a two-state photon portal for genuinely separate branes. The final sections set out stability conditions, numerical equations and decisive failure criteria.

Central limitation. The framework does not constitute a unique derivation of the website's claims. It is a concrete model that can be falsified. A different action could implement the same verbal ideas and predict different observations.

Contents

Reading guide and status labels

Label	Meaning
Established framework	Equation taken from an existing, published mathematical theory, with its assumptions stated.
Chosen ansatz	A modelling choice that is not forced by the website or by established physics.
Derived result	Algebraic or variational consequence of equations already stated in this document.
Closure requirement	Additional information needed because lower-dimensional equations do not determine the higher-dimensional dynamics.
Failure criterion	An observation or internal inconsistency that would reject the displayed version of the model.

The document uses metric signature $(-,+,+,+)$. Greek indices μ,ν run over four-dimensional spacetime; capital Latin indices A,B run over the five-dimensional bulk. Sections on brane projection and cosmology normally use $c=1$. Factors of c are restored in observational equations.

Part I Core model and its exact consequences

1 Mathematical target

The website states that galaxies inhabit broad curvature basins, membrane tension drives accelerated and directional expansion, and light may cross soft boundaries between neighbouring universe-regions [1]. To become a calculable model, those statements require at least:

1. a field variable and source equation for the curvature basin;
2. a relativistic metric that governs both matter and light;
3. a five-dimensional action and boundary conditions for the membrane;
4. a background cosmological solution and perturbation equations;
5. a stress source for directional expansion;
6. electromagnetic matching or a portal that permits photons to cross a boundary;
7. conservation, stability and falsification conditions.

This document adopts the following organising identity:

$$\boxed{a_\sigma(t) = \eta c^2 \sqrt{\frac{\Lambda_{\mathrm{eff}}(t)}{3}} = \eta c H_\sigma(t)} \tag{1.1}$$

Here a_σ is the acceleration below which extended curvature becomes important, η is one universal dimensionless coupling and Λ_{eff} is the part of the cosmological curvature attributed to tension or a strain potential. Equation (1.1) is the proposed bridge between galactic and cosmological scales. It is an ansatz until derived from the full action.

2 Nonlinear curvature equation

2.1 Chosen field equation

Let Φ be the dimensional gravitational potential and ρ_b the observed baryonic mass density. The core non-relativistic equation is

$$\boxed{\nabla \cdot \left[\frac{|\nabla\Phi|}{\sqrt{|\nabla\Phi|^2 + a_\sigma^2}} \nabla\Phi \right] = 4\pi G \rho_\mathrm{b}.} \tag{2.1}$$

Define

$$x = \frac{|\nabla\Phi|}{a_\sigma}, \qquad \mu_A(x) = \frac{x}{\sqrt{1+x^2}}. \tag{2.2}$$

Then Equation (2.1) is $\nabla \cdot [\mu_A(x)\nabla\Phi] = 4\pi G\rho_{\mathrm{b}}$. It is an explicit member of the AQUAL family introduced by Bekenstein and Milgrom [2]. Its use here is therefore not itself original.

2.2 Action and variation

The equation follows from

$$S_\Phi = -\int \mathrm{d}t\ \mathrm{d}^3x \left[\frac{a_\sigma^2}{8\pi G} F\left(\frac{|\nabla\Phi|^2}{a_\sigma^2}\right) + \rho_{\mathrm{b}}\Phi\right], \tag{2.3}$$

where

$$F(y) = \sqrt{y(1+y)} - \operatorname{arsinh}\sqrt{y}, \qquad F_y = \sqrt{\frac{y}{1+y}}. \tag{2.4}$$

Varying $\Phi \mapsto \Phi + \delta\Phi$, integrating by parts and setting the boundary variation to zero gives

$$\delta S_\Phi = \int \mathrm{d}t\ \mathrm{d}^3x \left[\frac{1}{4\pi G}\nabla\cdot(F_y\nabla\Phi) - \rho_{\mathrm{b}}\right]\delta\Phi, \tag{2.5}$$

$$\frac{\delta S_\Phi}{\delta\Phi} = 0 \quad \Longrightarrow \quad \nabla\cdot(F_y\nabla\Phi) = 4\pi G\rho_{\mathrm{b}}. \tag{2.6}$$

The limiting forms are

$$F(y) = \frac{2}{3}y^{3/2} - \frac{1}{5}y^{5/2} + \mathcal{O}(y^{7/2}), \qquad y \ll 1, \tag{2.7}$$

$$F(y) = y - \ln(2\sqrt{y}) + \frac{1}{2} - \frac{3}{8y} + \mathcal{O}(y^{-2}), \qquad y \gg 1. \tag{2.8}$$

Thus the deep-curvature action scales as $|\nabla\Phi|^3$, while its derivative approaches the Newtonian response at high acceleration.

3 Spherical solution and galaxy predictions

For a spherical baryonic mass $M_{\mathrm{b}}(r)$, define

$$g(r) = |\nabla\Phi|, \qquad g_{\mathrm{N}}(r) = \frac{GM_{\mathrm{b}}(r)}{r^2}. \tag{3.1}$$

Gauss' theorem gives

$$\frac{g^2}{\sqrt{g^2 + a_\sigma^2}} = g_{\mathrm{N}}. \tag{3.2}$$

The positive solution is

$$\boxed{g(r) = \left[\frac{g_{\mathrm{N}}^2 + \sqrt{g_{\mathrm{N}}^4 + 4a_\sigma^2 g_{\mathrm{N}}^2}}{2}\right]^{1/2}}. \tag{3.3}$$

Writing $y = g_N/a_\sigma$ gives the algebraic spherical interpolation

$$g = \nu(y) g_N, \qquad \nu(y) = \left[\frac{1 + \sqrt{1 + 4/y^2}}{2}\right]^{1/2}. \tag{3.4}$$

This algebraic relation is exact only under spherical symmetry. For a disc or interacting system, Equation (2.1) must be solved as a nonlinear partial differential equation because a non-zero curl field can occur.

3.1 High-acceleration limit

For $g_N \gg a_\sigma$,

$$g = g_N \left[1 + \frac{1}{2}\left(\frac{a_\sigma}{g_N}\right)^2 + \mathcal{O}\left(\frac{a_\sigma^4}{g_N^4}\right)\right]. \tag{3.5}$$

The leading anomalous acceleration is therefore

$$\delta g \simeq \frac{a_\sigma^2}{2g_N}. \tag{3.6}$$

This falls rapidly in high-acceleration environments, but Solar-System and binary tests must still be applied to the relativistic completion rather than only to this spherical expression.

3.2 Deep-curvature limit

For $g_N \ll a_\sigma$,

$$g \simeq \sqrt{g_N a_\sigma} = \frac{\sqrt{GM_b a_\sigma}}{r}. \tag{3.7}$$

The far-field potential and asymptotic circular speed are

$$\Phi(r) \simeq \sqrt{GM_b a_\sigma}\, \ln\left(\frac{r}{r_0}\right), \tag{3.8}$$

$$\boxed{v_\infty^4}\boxed{= GM_b a_\sigma}. \tag{3.9}$$

The transition radius is

$$r_\sigma = \sqrt{\frac{GM_b}{a_\sigma}}. \tag{3.10}$$

Direct test. A universal a_σ predicts the same normalisation $v_\infty^4/(GM_b)$ for isolated galaxies. A separate dark-halo mass is not available as a per-galaxy fitting parameter.

4 Effective curvature density

The same potential can be rewritten as a Newtonian Poisson equation with an effective geometric density:

$$\nabla^2\Phi = 4\pi G(\rho_{\mathrm{b}} + \rho_{\mathrm{curv}}), \tag{4.1}$$

$$\rho_{\mathrm{curv}} = \frac{1}{4\pi G}\nabla\cdot\left[(1-\mu_A)\nabla\Phi\right]. \tag{4.2}$$

This is a mathematical re-expression, not a new material substance. For an isolated spherical source outside the baryons in the deep regime,

$$M_{\mathrm{eff}}(r) = \frac{r^2 g}{G} = \sqrt{\frac{M_{\mathrm{b}}a_\sigma}{G}}\,r, \tag{4.3}$$

$$\rho_{\mathrm{curv}}(r) = \frac{1}{4\pi r^2}\sqrt{\frac{M_{\mathrm{b}}a_\sigma}{G}}. \tag{4.4}$$

The effective mass grows linearly with radius and the geometric density falls as r^{-2}. A finite environment or cosmological boundary condition is therefore required to truncate the logarithmic potential.

5 External-field response

The nonlinear equation violates the strong equivalence principle in the same way as AQUAL: a constant external field changes the internal response. Let

$$\nabla\Phi = \boldsymbol{g}_{\mathrm{e}} + \nabla\phi, \qquad |\nabla\phi| \ll g_{\mathrm{e}}. \tag{5.1}$$

Linearising gives

$$\partial_i\left[\mu_{\mathrm{e}}\left(\delta_{ij} + L_{\mathrm{e}}\hat{e}_i\hat{e}_j\right)\partial_j\phi\right] = 4\pi G\rho_{\mathrm{int}}, \tag{5.2}$$

where

$$\mu_{\mathrm{e}} = \mu_A(g_{\mathrm{e}}/a_\sigma), \qquad L_{\mathrm{e}} = \left.\frac{\mathrm{d}\ln\mu_A}{\mathrm{d}\ln x}\right|_{x=g_{\mathrm{e}}/a_\sigma} = \frac{1}{1+(g_{\mathrm{e}}/a_\sigma)^2}. \tag{5.3}$$

For a Fourier mode at angle ϑ to the external field,

$$G_{\mathrm{eff}}(\vartheta) = \frac{G}{\mu_{\mathrm{e}}\left(1 + L_{\mathrm{e}}\cos^2\vartheta\right)}. \tag{5.4}$$

This predicts an anisotropic internal potential in satellites, wide binaries and open clusters. It is not optional once Equation (2.1) is adopted.

Part II Five-dimensional membrane geometry

6 Bulk and brane action

A literal membrane requires a higher-dimensional spacetime. Let X^A be coordinates in a five-dimensional bulk $\mathcal{M}$, and let the four-dimensional brane $\mathcal{B}$ be embedded as $X^A(x^\mu)$. The induced metric is

$$g_{\mu\nu} = G_{AB} e^A{}_\mu e^B{}_\nu, \qquad e^A{}_\mu = \frac{\partial X^A}{\partial x^\mu}. \tag{6.1}$$

With spacelike unit normal n^A, the extrinsic curvature is

$$K_{\mu\nu} = e^A{}_\mu e^B{}_\nu \nabla_A n_B. \tag{6.2}$$

A minimal action is

$$\begin{aligned} S = \frac{1}{2\kappa_5^2} \int_{\mathcal{M}} \mathrm{d}^5 X \sqrt{-G}\,(R_5 - 2\Lambda_5) + \frac{1}{\kappa_5^2} \int_{\mathcal{B}} \mathrm{d}^4 x \sqrt{-g}\,[K] \\ + \int_{\mathcal{B}} \mathrm{d}^4 x \sqrt{-g}\,[-\sigma + \mathcal{L}_{\mathrm{b}} + \mathcal{L}_{\mathrm{mem}}]\,. \end{aligned} \tag{6.3}$$

Here σ is the fundamental brane tension, $\mathcal{L}_{\mathrm{b}}$ is baryonic matter, and $\mathcal{L}_{\mathrm{mem}}$ is an additional closure sector introduced later. The boundary term gives a well-defined metric variation. This is the standard starting point of brane gravity, not a derivation from the website [5, 3].

7 Junction condition

Let the total brane stress tensor be

$$S_{\mu\nu} = -\sigma g_{\mu\nu} + T_{\mu\nu}. \tag{7.1}$$

The Israel junction condition is [6]

$$\boxed{[K_{\mu\nu}] = -\kappa_5^2 \left(S_{\mu\nu} - \frac{1}{3} S g_{\mu\nu} \right).} \tag{7.2}$$

For a Z_2-symmetric embedding, $K^+_{\mu\nu} = -K^-_{\mu\nu}$, so the extrinsic curvature on either side is fixed algebraically by $S_{\mu\nu}$. The junction condition converts membrane tension and matter into geometric bending.

8 Projected Einstein equation

Projecting the five-dimensional equations onto the brane gives the Shiromizu-Maeda-Sasaki equation [3]:

$$\boxed{G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + 8\pi G\, T_{\mu\nu} + \kappa_5^4 \Pi_{\mu\nu} - E_{\mu\nu}.} \tag{8.1}$$

For a vacuum bulk apart from Λ_5,

$$\Lambda_4 = \frac{\kappa_5^2}{2}\left(\Lambda_5 + \frac{\kappa_5^2\sigma^2}{6}\right), \tag{8.2}$$

$$8\pi G = \frac{\kappa_5^4\sigma}{6}, \tag{8.3}$$

$$\Pi_{\mu\nu} = -\frac{1}{4}T_{\mu\alpha}T_\nu{}^\alpha + \frac{1}{12}TT_{\mu\nu} + \frac{1}{8}g_{\mu\nu}T_{\alpha\beta}T^{\alpha\beta} - \frac{1}{24}g_{\mu\nu}T^2, \tag{8.4}$$

$$E_{\mu\nu} = C^{(5)}{}_{ABCD}n^A n^C e^B{}_\mu e^D{}_\nu. \tag{8.5}$$

$E_{\mu\nu}$ is the projected bulk Weyl tensor. It is trace-free and represents nonlocal gravitational information from the bulk. The brane equation is not closed because $E_{\mu\nu}$ cannot generally be calculated from brane data alone.

The Bianchi identity gives, when ordinary matter is conserved,

$$\nabla^\mu E_{\mu\nu} = \kappa_5^4 \nabla^\mu \Pi_{\mu\nu}. \tag{8.6}$$

This is a constraint, not an evolution equation sufficient to determine all components of $E_{\mu\nu}$.

Closure problem. Calling $E_{\mu\nu}$ "extended curvature" does not calculate it. A bulk solution, boundary condition or effective constitutive law is required. Without one, rotation curves and lensing are not predictions of the five-dimensional theory.

9 Tension, cosmological curvature and Newton's constant

Equations (8.2) and (8.3) expose an important consistency condition. If the fundamental brane tension varies while κ_5 and Λ_5 are fixed, then

$$\frac{\dot{G}}{G} = \frac{\dot{\sigma}}{\sigma}, \tag{9.1}$$

$$\dot{\Lambda}_4 = 8\pi G\,\dot{\sigma}. \tag{9.2}$$

A time-varying fundamental tension therefore changes both the cosmological term and the strength of gravity. The clean construction used later keeps σ fixed and introduces a separate dynamical strain field whose potential energy behaves like an effective tension.

Part III Candidate relativistic closure and lensing

10 Why a closure field is introduced

The projected Weyl tensor may be viewed as a geometric dark sector, but a brane observer still needs local equations to evolve it. One possible effective representation uses a unit timelike vector U^μ that defines the local rest frame of the membrane response. This has precedents in relativistic MOND and generalised Einstein-aether models [7, 8, 9].

The four-dimensional vector U^μ is not the spacelike five-dimensional normal n^A. It is an additional brane degree of freedom, or an effective representation of bulk dynamics. A theory that insists on no additional effective field must instead solve the full five-dimensional bulk directly.

11 Covariant strain action

Impose

$$U^\mu U_\mu = -1, \qquad a_\mu = U^\nu \nabla_\nu U_\mu, \qquad Y = \frac{a_\mu a^\mu}{M^2}, \qquad M = \frac{a_\sigma}{c^2}. \tag{11.1}$$

A candidate effective four-dimensional action is

$$S_4 = \frac{c^4}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left[R - 2\Lambda_{\text{eff}} - 2M^2\mathcal{G}(Y) + \lambda(U^\mu U_\mu + 1) + \mathcal{L}_{\text{stab}}\right] + S_{\text{b}}[g_{\mu\nu}, \psi_{\text{b}}], \tag{11.2}$$

where $\mathcal{L}_{\text{stab}}$ contains the additional derivative combinations needed to make vector and scalar perturbations healthy. The matching function is

$$\mathcal{G}(Y) = F(Y) - Y = \sqrt{Y(1+Y)} - \operatorname{arsinh}\sqrt{Y} - Y. \tag{11.3}$$

In a static weak field with U^μ aligned to the timelike Killing direction,

$$a_i = \frac{\partial_i \Phi}{c^2} + \mathcal{O}(c^{-4}), \qquad Y = \frac{|\nabla\Phi|^2}{a_\sigma^2} + \mathcal{O}(c^{-2}). \tag{11.4}$$

The Einstein-Hilbert term supplies the ordinary weak-field piece proportional to Y, while $\mathcal{G} = F - Y$ changes the total to $F(Y)$. After solving the weak-field metric constraints, the reduced action is Equation (2.3).

Chosen ansatz. Equation (11.2) demonstrates how a covariant effective action can be arranged to reproduce the galaxy equation. It is not yet a complete theory because $\mathcal{L}_{\text{stab}}$, the bulk interpretation and all mode speeds must be specified and tested.

The field equations are defined by

$$G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu} = \frac{8\pi G}{c^4} T^{\text{b}}_{\mu\nu} + T^{(U)}_{\mu\nu}, \tag{11.5}$$

$$T^{(U)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_U}{\delta g^{\mu\nu}}, \qquad \frac{1}{\sqrt{-g}}\frac{\delta S_4}{\delta U^\mu} = 0, \qquad U^2 = -1. \tag{11.6}$$

In a genuine brane completion, $T^{(U)}_{\mu\nu}$ is replaced by, or matched to, $-E_{\mu\nu} + \kappa_5^4 \Pi_{\mu\nu}$.

12 Relativistic metric and gravitational slip

Write the weak static metric as

$$ds^2 = -\left(1 + \frac{2\Phi}{c^2}\right) c^2 \, dt^2 + \left(1 - \frac{2\Psi}{c^2}\right) d\boldsymbol{x}^2. \tag{12.1}$$

Slow matter responds mainly to Φ. Light responds to $\Phi + \Psi$. Define the slip

$$\gamma_{\text{slip}} = \frac{\Psi}{\Phi}. \tag{12.2}$$

To make the same curvature explain both rotation and lensing without an independent halo, the static closure must produce approximately

$$\boxed{\Psi = \Phi \quad \text{on galaxy and cluster lensing scales.}} \tag{12.3}$$

This occurs in some relativistic modified-gravity constructions, but it is not guaranteed merely by Equation (2.1) [7].

13 Lensing equations

The weak deflection angle for a ray with unperturbed line-of-sight coordinate ℓ is

$$\boldsymbol{\alpha}(\boldsymbol{b}) = \frac{1}{c^2} \int_{-\infty}^{\infty} \nabla_\perp (\Phi + \Psi) \, d\ell. \tag{13.1}$$

Under the no-slip condition,

$$\boldsymbol{\alpha}(\boldsymbol{b}) = \frac{2}{c^2} \int \nabla_\perp \Phi \, d\ell. \tag{13.2}$$

For the deep-curvature logarithmic potential, set $V^2 = \sqrt{GM_{\text{b}} a_\sigma}$ and $r^2 = b^2 + \ell^2$. Then

$$\frac{\partial \Phi}{\partial b} = \frac{V^2 b}{b^2 + \ell^2}, \tag{13.3}$$

$$\alpha(b) = \frac{2V^2}{c^2} \int_{-\infty}^{\infty} \frac{b \, d\ell}{b^2 + \ell^2} = \boxed{\frac{2\pi}{c^2} \sqrt{GM_{\text{b}} a_\sigma}}. \tag{13.4}$$

The asymptotic deflection is independent of impact parameter until the curvature basin is truncated by its environment.

For a thin lens,

$$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_{\text{s}}}{D_{\text{l}} D_{\text{ls}}}, \tag{13.5}$$

$$\kappa(\boldsymbol{\theta}) = \frac{\Sigma_{\text{eff}}(D_{\text{l}} \boldsymbol{\theta})}{\Sigma_{\text{crit}}}, \tag{13.6}$$

$$\Sigma_{\text{eff}}(b) = \int (\rho_{\text{b}} + \rho_{\text{curv}}) \, d\ell. \tag{13.7}$$

For Equation (4.4),

$$\Sigma_{\text{curv}}(b) = \frac{V^2}{4Gb}, \qquad \theta_{\text{E}} = \frac{D_{\text{ls}}}{D_{\text{s}}} \frac{2\pi V^2}{c^2}. \tag{13.8}$$

These are direct lensing predictions once the baryonic mass, a_σ and truncation boundary condition are fixed. Brane-world lensing has been studied previously and can differ sharply from particle-halo lensing [10].

14 Clusters and the need for dynamical curvature

The equilibrium AQUAL equation is elliptic. Its effective curvature density follows the current baryonic distribution. Merging clusters therefore provide a stringent test because observed lensing peaks can be displaced from the dominant hot-gas baryons [11].

A minimal optional bulk-memory closure is

$$\nabla^2\Phi = 4\pi G(\rho_\mathrm{b} + \rho_W), \tag{14.1}$$

$$\tau_W^2\left(\ddot\rho_W + 3H\dot\rho_W\right) - \ell_W^2\nabla^2\rho_W + \rho_W = \rho^\mathrm{eq}_\mathrm{curv}[\rho_\mathrm{b}], \tag{14.2}$$

where ρ_W represents a dynamical projection of bulk Weyl curvature and $\rho^\mathrm{eq}_\mathrm{curv}$ is Equation (4.2). The propagation speed is $c_W = \ell_W/\tau_W$.

Equation (14.2) is not part of the minimal galaxy model. It shows what must be added if geometric curvature is to retain memory and become spatially displaced during a merger. Its parameters cannot be introduced solely to fit one cluster; they must follow from the bulk action and work across all systems.

Part IV Cosmological background and structure formation

15 Brane Friedmann equation

For a homogeneous and isotropic brane,

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\,\mathrm{d}\Omega^2\right]. \tag{15.1}$$

The brane Friedmann equation is [4]

$$\boxed{H^2 = \frac{\Lambda_4}{3} + \frac{8\pi G}{3}\rho\left(1+\frac{\rho}{2\sigma}\right) + \frac{\mathcal{C}}{a^4} - \frac{k}{a^2}.} \tag{15.2}$$

$\mathcal{C}/a^4$ is the dark-radiation integration term arising from the bulk Weyl tensor. Standard expansion is recovered at $\rho/\sigma \ll 1$ if $\mathcal{C}$ is small.

With conserved brane matter,

$$\dot\rho + 3H(\rho + p) = 0. \tag{15.3}$$

Differentiating Equation (15.2) gives

$$\frac{\ddot{a}}{a} = \frac{\Lambda_4}{3} - \frac{4\pi G}{3}\left[(\rho + 3p) + \frac{\rho}{\sigma}(2\rho + 3p)\right] - \frac{\mathcal{C}}{a^4}. \tag{15.4}$$

A constant positive Λ_4 accelerates expansion. The high-energy ρ^2 term and positive dark radiation are decelerating for ordinary matter.

16 Why variable fundamental tension is problematic

If $\sigma = \sigma(t)$, conservation of the total brane stress in the absence of bulk exchange gives

$$\dot{\rho} + 3H(\rho + p) = -\dot{\sigma}. \tag{16.1}$$

Together with Equations (9.1) and (9.2), a changing σ simultaneously transfers energy, changes G and changes Λ_4. A variable fundamental tension is therefore not equivalent to an isolated dark-energy component.

17 Dynamical strain field

A cleaner model keeps the fundamental tension σ fixed and introduces a brane strain scalar q:

$$S_q = -\int d^4x\sqrt{-g}\left[\frac{1}{2}\nabla_\mu q \nabla^\mu q + V(q)\right]. \tag{17.1}$$

For a homogeneous field,

$$\rho_q = \frac{1}{2}\dot{q}^2 + V(q), \tag{17.2}$$

$$p_q = \frac{1}{2}\dot{q}^2 - V(q), \tag{17.3}$$

$$\ddot{q} + 3H\dot{q} + V_{,q} = 0, \tag{17.4}$$

and

$$w_q = \frac{p_q}{\rho_q}. \tag{17.5}$$

Define the tension-like cosmological sector by

$$H_\sigma^2(t) = \frac{\Lambda_4}{3} + \frac{8\pi G}{3}V(q), \tag{17.6}$$

$$a_\sigma(t) = \eta c H_\sigma(t) = \eta c^2\sqrt{\frac{1}{3}\left(\Lambda_4 + \frac{8\pi G}{c^4}V(q)\right)}. \tag{17.7}$$

A constant q with $V > 0$ behaves like vacuum energy. A rolling q allows time dependence without forcing the fundamental G to vary.

High-redshift prediction. If $V(q)$ evolves, the normalisation of the baryonic Tully-Fisher relation evolves:

$$\frac{v_\infty^4(z)}{GM_\mathrm{b}} = a_\sigma(z), \qquad \frac{a_\sigma(z)}{a_\sigma(0)} = \left[\frac{\Lambda_4 + 8\pi G V(q(z))/c^4}{\Lambda_4 + 8\pi G V(q_0)/c^4}\right]^{1/2}.$$

The same evolution must fit both galaxy dynamics and the expansion history.

18 Dimensionless expansion history

Let $\rho_{c0} = 3H_0^2/(8\pi G)$ and $\Omega_\sigma = \sigma/\rho_{c0}$. A convenient fitting form is

$$E^2(z) \equiv \frac{H^2(z)}{H_0^2} = \Omega_k(1+z)^2 + \Omega_{\Lambda_4} + \Omega_{\mathcal{C}}(1+z)^4 + \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_q f_q(z) + \frac{\left[\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_q f_q(z)\right]^2}{2\Omega_\sigma}, \tag{18.1}$$

where

$$f_q(z) = \exp\left[3\int_0^z \frac{1+w_q(z')}{1+z'}\,\mathrm{d}z'\right]. \tag{18.2}$$

The comoving and luminosity distances are

$$\chi(z) = c\int_0^z \frac{\mathrm{d}z'}{H(z')}, \tag{18.3}$$

$$d_L(z) = (1+z)S_k[\chi(z)], \tag{18.4}$$

with $S_k(\chi) = \chi$ in a spatially flat model. Equations (18.1)–(18.4) are the minimum background equations for comparison with expansion data.

19 Linear perturbations

Use conformal Newtonian gauge,

$$\mathrm{d}s^2 = a^2(\tau)\left[-(1+2\Psi)\,\mathrm{d}\tau^2 + (1-2\Phi)\,\mathrm{d}\boldsymbol{x}^2\right]. \tag{19.1}$$

The galaxy interpolation μ_A and the linear cosmological response must not be conflated. Define two scale- and time-dependent linear functions:

$$-k^2\Psi = 4\pi G a^2 \mu_L(k,a)\rho_m\Delta_m, \tag{19.2}$$

$$\Phi = \gamma(k,a)\Psi, \tag{19.3}$$

$$-k^2(\Phi+\Psi) = 8\pi G a^2 \Sigma(k,a)\rho_m\Delta_m, \qquad \Sigma = \frac{\mu_L(1+\gamma)}{2}. \tag{19.4}$$

This is the standard phenomenological separation between dynamical and lensing responses used in modified-gravity calculations [12].

On sub-horizon scales, a pressureless growth factor $D(a)$ approximately obeys

$$\frac{\mathrm{d}^2 D}{\mathrm{d}(\ln a)^2} + \left[2 + \frac{\mathrm{d}\ln H}{\mathrm{d}\ln a}\right]\frac{\mathrm{d}D}{\mathrm{d}\ln a} - \frac{3}{2}\Omega_m(a)\mu_L(k,a)D = 0. \tag{19.5}$$

The canonical strain perturbation satisfies

$$\delta q'' + 2\mathcal{H}\delta q' + (k^2 + a^2 V_{,qq})\delta q = q'(\Psi' + 3\Phi') - 2a^2 V_{,q}\Psi, \tag{19.6}$$

where primes denote conformal-time derivatives and $\mathcal{H} = a'/a$.

The integrated Sachs-Wolfe contribution is

$$\left.\frac{\Delta T}{T}\right|_{\mathrm{ISW}} = \int_{\tau_*}^{\tau_0} (\Phi' + \Psi')\, \mathrm{d}\tau. \tag{19.7}$$

CMB lensing is likewise governed by the line-of-sight integral of $\Phi + \Psi$.

19.1 The pressureless-mode requirement

Before recombination, a baryon-only universe does not retain the same potential wells as a cold, pressureless component. A no-particle model must therefore supply a geometric perturbation that clusters approximately as

$$w_{\mathrm{geom}} \simeq 0, \qquad c^2_{s,\mathrm{geom}} \ll 1, \qquad -k^2\Psi \supset 4\pi G a^2 \rho_{\mathrm{geom}}\Delta_{\mathrm{geom}}. \tag{19.8}$$

Relativistic MOND theories can generate additional growing modes, but the perturbation sector must be solved explicitly [13]. A component satisfying Equation (19.8) may be geometric rather than particulate, but observationally it acts as a dark clustering sector.

20 Projected Weyl fluid

For cosmological calculations, decompose the projected bulk Weyl tensor relative to a brane four-velocity u^μ:

$$E_{\mu\nu} = -\mathcal{A}\left[\rho_E\left(u_\mu u_\nu + \frac{1}{3}h_{\mu\nu}\right) + 2q^E_{(\mu}u_{\nu)} + \pi^E_{\mu\nu}\right], \tag{20.1}$$

where $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ and $\mathcal{A}$ is a convention-dependent normalisation. ρ_E, q^E_μ and $\pi^E_{\mu\nu}$ behave as a nonlocal effective fluid. Equation (8.6) supplies some evolution constraints but not a local equation for $\pi^E_{\mu\nu}$. A bulk calculation or closure such as Equation (14.2) remains necessary.

Part V Directional expansion

21 Bianchi-I background

A homogeneous anisotropic metric can be written

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\sum_{i=1}^{3} \mathrm{e}^{2\beta_i(t)}(\,\mathrm{d}x^i)^2, \qquad \sum_i \beta_i = 0. \tag{21.1}$$

Define

$$H_i = H + \dot{\beta}_i, \qquad H = \frac{1}{3}\sum_i H_i, \qquad \sigma_{\rm sh}^2 = \frac{1}{2}\sum_i \dot{\beta}_i^2. \tag{21.2}$$

The background equations are

$$3H^2 = 8\pi G\rho + \Lambda_{\rm eff} + \sigma_{\rm sh}^2, \tag{21.3}$$
$$\ddot{\beta}_i + 3H\dot{\beta}_i = 8\pi G\pi_i, \qquad \sum_i \pi_i = 0, \tag{21.4}$$

where π_i is anisotropic stress. If $\pi_i = 0$,

$$\dot{\beta}_i = \frac{C_i}{a^3}, \qquad \sigma_{\rm sh}^2 \propto a^{-6}. \tag{21.5}$$

Directional expansion therefore decays rapidly unless the membrane supplies sustained anisotropic stress.

22 A consistent tension-gradient source

A position-dependent pure vacuum term $T_{\mu\nu}^{(\sigma)} = -\sigma(x)g_{\mu\nu}$ is not independently conserved:

$$\nabla^\mu T_{\mu\nu}^{(\sigma)} = -\partial_\nu\sigma. \tag{22.1}$$

Thus a tension gradient requires energy exchange or a dynamical field. For the strain scalar q,

$$T_{\mu\nu}^{(q)} = \nabla_\mu q\nabla_\nu q - g_{\mu\nu}\left[\frac{1}{2}(\nabla q)^2 + V(q)\right], \tag{22.2}$$

and a coherent spatial gradient produces

$$\pi_{ij}^{(q)} = D_i q D_j q - \frac{1}{3}h_{ij}(Dq)^2. \tag{22.3}$$

Equations (21.4) and (22.3) provide a mathematically consistent route from a membrane strain gradient to directional expansion.

23 Observable directional Hubble law

For a source viewed along unit direction n^i, the local directional rate is

$$H(\boldsymbol{n}) = \sum_i n_i^2 H_i = H + \sum_i n_i^2\dot{\beta}_i. \tag{23.1}$$

At low redshift,

$$d_L(z,\boldsymbol{n}) = \frac{cz}{H(\boldsymbol{n})} + \mathcal{O}(z^2). \tag{23.2}$$

Equivalently,

$$\frac{\delta H(\boldsymbol{n})}{H} = Q_{ij}n^i n^j, \qquad Q_{ij} = \frac{\sigma_{ij}}{H}, \qquad Q^i{}_i = 0. \tag{23.3}$$

Angular signature. A homogeneous Bianchi-I strain produces a quadrupolar Hubble anisotropy, not an arbitrary dipole. A dominant dipole would require an inhomogeneous boundary, an off-centre observer or peculiar motion in addition to homogeneous shear.

Part VI Boundary regions and photon transfer

24 Geometric interface

Model a soft boundary as a thin effective timelike hypersurface Σ separating four-dimensional regions $\mathcal{M}_-$ and $\mathcal{M}_+$. Let h_{ab} be its induced metric and $\mathscr{K}^{\pm}_{ab}$ its extrinsic curvature as embedded in each region. The thin-wall matching equations are

$$[h_{ab}] = 0, \tag{24.1}$$

$$[\mathscr{K}_{ab}] - h_{ab}[\mathscr{K}] = -\frac{8\pi G}{c^4} S^{\Sigma}_{ab}. \tag{24.2}$$

A genuinely soft boundary replaces the surface tensor by a finite-thickness stress profile and requires solving the field equations across that layer. The thin-wall equations are the controlled limiting case.

25 Maxwell matching

Let the electromagnetic kinetic coefficient be $Z_\pm$ in the two regions:

$$S_{\mathrm{EM}} = -\frac{1}{4}\sum_{s=\pm}\int_{\mathcal{M}_s} \mathrm{d}^4x\sqrt{-g_s}\, Z_s F^{(s)}_{\mu\nu} F^{\mu\nu}_{(s)} - \int_{\Sigma} \mathrm{d}^3y\sqrt{-h}\, A_a j^a_{\Sigma}. \tag{25.1}$$

The field equation and boundary conditions are

$$\nabla_\mu(ZF^{\mu\nu}) = J^\nu, \tag{25.2}$$

$$[F_{ab}] = 0, \tag{25.3}$$

$$n_\mu[ZF^{\mu a}] = j^a_{\Sigma}. \tag{25.4}$$

With no surface current, tangential electric and magnetic fields match and the normal flux weighted by Z is continuous.

At normal incidence between lossless regions with wave impedances $\mathcal{Z}_-$ and $\mathcal{Z}_+$,

$$\mathcal{R} = \left(\frac{\mathcal{Z}_+ - \mathcal{Z}_-}{\mathcal{Z}_+ + \mathcal{Z}_-}\right)^2, \tag{25.5}$$

$$\mathcal{T}=\frac{4\mathcal{Z}_+\mathcal{Z}_-}{(\mathcal{Z}_++\mathcal{Z}_-)^2},\qquad \mathcal{R}+\mathcal{T}=1. \tag{25.6}$$

A transparent boundary requires $\mathcal{Z}_+\simeq\mathcal{Z}_-$ or another portal mechanism.

26 Geometric optics and redshift

For an eikonal phase Θ,

$$k_\mu=\nabla_\mu\Theta,\qquad k^\mu k_\mu=0,\qquad k^\nu\nabla_\nu k^\mu=0. \tag{26.1}$$

Phase continuity at the interface implies continuity of tangential wavevector components:

$$h_a{}^\mu[k_\mu]=0. \tag{26.2}$$

An observer of four-velocity u^μ measures $\omega=-u^\mu k_\mu$. Let U^μ_Σ be the interface velocity. If the interface is stationary in its own frame so that $\omega^-_\Sigma=\omega^+_\Sigma$, the total redshift from an emitter in $\mathcal{M}_-$ to an observer in $\mathcal{M}_+$ is

$$\boxed{1+z_{\rm obs}=\frac{-u_{\rm e}\cdot k_-}{-U_\Sigma\cdot k_-}\,\frac{-U_\Sigma\cdot k_+}{-u_{\rm o}\cdot k_+}.} \tag{26.3}$$

This replaces the ordinary single-region formula. A proposed boundary explanation of a high apparent redshift must calculate all four contractions, not merely assert that the photon was bent.

27 Flux, surface brightness and the JWST claim

For a source luminosity $L_{\rm e}$, boundary transmission $\mathcal{T}$ and net geometric magnification $\mathcal{M}_\Sigma$,

$$F_{\rm o}=\frac{\mathcal{T}\,\mathcal{M}_\Sigma\,L_{\rm e}}{4\pi d_L^2}. \tag{27.1}$$

For metric propagation with photon number conserved along each leg, distance duality gives

$$d_L=(1+z_{\rm obs})^2d_A. \tag{27.2}$$

Bolometric surface brightness transforms as

$$I_{\rm o}=\frac{\mathcal{T}}{(1+z_{\rm obs})^4}I_{\rm e}. \tag{27.3}$$

Important consequence. Passive transmission has $\mathcal{T}\le 1$ and makes a source dimmer. Boundary crossing by itself cannot make a galaxy appear anomalously luminous or massive. The model would need geometric magnification, a different distance-redshift mapping, or both.

A non-dispersive metric boundary predicts that every identified spectral feature gives the same redshift. Define

$$z_i=\frac{\lambda_{i,\rm obs}}{\lambda_{i,\rm rest}}-1,\qquad \Delta z_{ij}=\frac{z_i-z_j}{1+\bar z}. \tag{27.4}$$

The minimal model predicts $\Delta z_{ij} = 0$ within measurement errors. A wavelength-dependent result requires explicit dispersion in $Z(\omega)$ or in the portal.

Temporal features must obey

$$\Delta t_{\mathrm{o}} = (1 + z_{\mathrm{obs}})\Delta t_{\mathrm{e}} \tag{27.5}$$

for a stationary, non-dispersive interface. The angular distribution of boundary-origin candidates can be parameterised as

$$N(\boldsymbol{n}, z) = N_0(z)\left[1 + A_1(z)\boldsymbol{n}\cdot\boldsymbol{d} + A_2(z)P_2(\boldsymbol{n}\cdot\boldsymbol{d}) + \cdots\right]. \tag{27.6}$$

A boundary explanation predicts coherent sky multipoles tied to the boundary direction, rather than an isotropic excess.

28 Separate-brane photon portal

If neighbouring universes are separate branes rather than adjacent domains in one four-dimensional spacetime, ordinary photons are not automatically able to leave their brane. A portal coupling is required. A minimal two-state propagation model is

$$i\frac{\mathrm{d}}{\mathrm{d}\lambda}\begin{pmatrix} A_+ \\ A_- \end{pmatrix} = \begin{pmatrix} \Delta_+ & \epsilon \\ \epsilon & \Delta_- \end{pmatrix}\begin{pmatrix} A_+ \\ A_- \end{pmatrix}. \tag{28.1}$$

Let $\Delta = \Delta_+ - \Delta_-$. Over affine length L, the transfer probability is

$$\boxed{P_{-\to+}(L) = \frac{4\epsilon^2}{\Delta^2 + 4\epsilon^2}\sin^2\left[\frac{L}{2}\sqrt{\Delta^2 + 4\epsilon^2}\right].} \tag{28.2}$$

Without a non-zero ϵ or an equivalent bulk electromagnetic field, $P_{-\to+} = 0$. The portal also risks producing oscillatory, polarisation-dependent or frequency-dependent signatures that must be searched for.

Part VII Mathematical consistency and computation

29 Ellipticity of the galaxy equation

The principal response matrix obtained by linearising Equation (2.1) about a field $\boldsymbol{g}$ is

$$A_{ij} = \mu_A\left(\delta_{ij} + L\hat{g}_i\hat{g}_j\right), \qquad L = \frac{\mathrm{d}\ln\mu_A}{\mathrm{d}\ln x} = \frac{1}{1+x^2}. \tag{29.1}$$

Its two transverse eigenvalues are μ_A and its longitudinal eigenvalue is $\mu_A(1+L)$. For $x > 0$,

$$\mu_A > 0, \qquad 1 + L > 0, \tag{29.2}$$

so the equation is elliptic. It becomes degenerately elliptic as $x \to 0$ because $\mu_A \to 0$.

The action has

$$F_y = \sqrt{\frac{y}{1+y}} > 0, \tag{29.3}$$

$$F_{yy} = \frac{1}{2\sqrt{y}(1+y)^{3/2}} > 0, \tag{29.4}$$

$$F_y + 2yF_{yy} = \frac{\sqrt{y}(y+2)}{(1+y)^{3/2}} > 0. \tag{29.5}$$

These conditions give a positive incremental susceptibility in the static equation. They do not replace the ghost and gradient-stability analysis of the full covariant theory.

30 Relativistic stability checklist

For the candidate vector closure, the following must all be demonstrated:

1. the Hamiltonian is bounded below around cosmological and local backgrounds;
2. scalar, vector and tensor kinetic matrices have positive eigenvalues;
3. squared propagation speeds are non-negative;
4. the physical characteristic cones are compatible with causality and high-energy particle constraints;
5. post-Newtonian preferred-frame parameters satisfy local tests;
6. the tensor speed is luminal to observational precision;
7. no caustic or gradient singularity forms in generic evolution.

In the conventional Einstein-aether notation, the tensor speed is

$$c_T^2 = \frac{c^2}{1-c_{13}}, \qquad c_{13} = c_1 + c_3. \tag{30.1}$$

The exact luminal condition in that convention is $c_{13} = 0$ [9]. The nonlinear acceleration term in Equation (11.2) may modify other mode speeds, which must be derived from the quadratic action.

31 Energy conservation

Diffeomorphism invariance requires

$$\nabla^\mu \left(T^{\mathrm{b}}_{\mu\nu} + T^{(U)}_{\mu\nu} + T^{(q)}_{\mu\nu} \right) = 0 \tag{31.1}$$

in a closed four-dimensional effective theory. In the brane description, apparent non-conservation can instead represent energy-momentum exchange with the bulk:

$$\nabla^\mu T_{\mu\nu} = Q_\nu, \qquad Q_\nu = -2T^{(5)}_{AB} n^A e^B{}_\nu. \tag{31.2}$$

Any changing tension, boundary leakage or photon portal must appear in a corresponding Q_ν and in the bulk stress budget.

32 Numerical galaxy solver

For an arbitrary baryonic density, define the residual

$$\mathcal{R}[\Phi] = \nabla \cdot \left[\mu_A\left(\frac{|\nabla\Phi|}{a_\sigma}\right)\nabla\Phi\right] - 4\pi G\rho_{\mathrm{b}}. \tag{32.1}$$

A Newton update $\Phi \mapsto \Phi + \delta\Phi$ solves

$$\partial_i\left[\mu_A\left(\delta_{ij} + L\hat{g}_i\hat{g}_j\right)\partial_j\delta\Phi\right] = -\mathcal{R}[\Phi]. \tag{32.2}$$

Useful boundary conditions are symmetry conditions at the origin or mid-plane and an environment-dependent outer condition. For an isolated spherical approximation,

$$\left.\frac{\partial\Phi}{\partial r}\right|_{r=R_{\mathrm{out}}} \simeq \frac{\sqrt{GM_{\mathrm{b}}a_\sigma}}{R_{\mathrm{out}}}, \tag{32.3}$$

but a finite external field should replace this for real galaxies.

33 Joint likelihood

Let $\boldsymbol{d}$ contain rotation velocities, lensing observables, distance measurements, growth data and anisotropy coefficients. For model prediction $\boldsymbol{m}(\boldsymbol{\theta})$ and covariance C,

$$-2\ln\mathcal{L}(\boldsymbol{\theta}) = [\boldsymbol{d} - \boldsymbol{m}(\boldsymbol{\theta})]^T C^{-1}[\boldsymbol{d} - \boldsymbol{m}(\boldsymbol{\theta})] + \ln\det C + \mathrm{const.} \tag{33.1}$$

A credible analysis must use one shared parameter vector. In particular, η, $a_\sigma(0)$, the bulk parameters and any portal or relaxation scales cannot be re-fitted independently for every observational class.

34 Minimal parameter hierarchy

Layer	**Parameters**	**Role**
Galaxy core	a_σ or $(\eta, \Lambda_{\mathrm{eff}})$	Fixes the transition and Tully-Fisher normalisation.
Five-dimensional background	$\kappa_5, \Lambda_5, \sigma, \mathcal{C}$	Sets G, Λ_4, high-energy corrections and dark radiation.
Strain field	Parameters of $V(q)$	Controls time evolution of effective tension and $a_\sigma(z)$.
Relativistic closure	Coefficients in $\mathcal{L}_{\mathrm{stab}}$	Fixes slip, mode speeds, post-Newtonian effects and perturbations.
Bulk-memory extension	τ_W, ℓ_W	Optional cluster-scale lag and smoothing.
Directional sector	Initial shear and strain-gradient amplitude	Sets the angular Hubble quadrupole.
Boundary optics	$\mathcal{Z}_+/\mathcal{Z}_-$ or (ϵ, Δ)	Sets photon reflection, transmission or inter-brane conversion.

The minimal version should set optional layers to zero until a failure of the simpler model requires them. Otherwise the framework becomes too flexible to falsify.

Part VIII Decisive predictions and failure criteria

35 Prediction set

A useful first publication would commit to the following linked predictions:

1. **Galaxy dynamics:** $v_\infty^4 = GM_\mathrm{b} a_\sigma$ with one universal present-day a_σ.
2. **Rotation-lensing identity:** the same Φ that fits rotation curves, with $\Psi \simeq \Phi$, predicts the lensing map without an independent halo.
3. **Cosmological link:** $a_\sigma(z) = \eta c^2 \sqrt{\Lambda_\mathrm{eff}(z)/3}$ with one universal η.
4. **Directional signature:** homogeneous membrane strain produces the quadrupole in Equation (23.3) and obeys the shear evolution Equation (21.4).
5. **Boundary spectroscopy:** all lines share one z_obs, time dilation follows Equation (27.5), and candidates have coherent sky multipoles.
6. **Photon-transfer budget:** the observed number and brightness of boundary candidates follow Equation (27.1) with $\mathcal{T}$ or Equation (28.2) with no hidden source term.

36 Failure criteria

The displayed version should be rejected or substantially revised if any of the following occurs:

1. no universal a_σ can jointly fit well-measured isolated galaxy rotation curves within baryonic uncertainties;
2. the relativistic completion fitting dynamics predicts the wrong lensing amplitude or morphology;
3. the required cluster offsets demand arbitrary system-specific memory parameters;
4. the background expansion and the inferred $a_\sigma(z)$ disagree under the shared identity Equation (1.1);
5. no stable pressureless geometric mode can reproduce large-scale structure and CMB gravitational potentials;
6. the vector or strain sector contains ghosts, gradient instabilities, non-luminal tensor propagation or unacceptable preferred-frame effects;
7. directional expansion lacks the angular pattern predicted by its stress tensor;
8. proposed boundary objects fail achromatic redshift, time-dilation, distance-duality or sky-correlation tests;
9. the portal probability required by the claimed population conflicts with electromagnetic leakage or spectral constraints elsewhere.

37 What is original and what is not

Component	Assessment
Nonlinear Poisson/AQUAL structure	Established modified-gravity mathematics, not original [2].
Five-dimensional brane projection and ρ^2 Friedmann term	Established brane-world mathematics [3, 4].
Vector or scalar relativistic completion	Existing families show how this can be done, although the exact matching used here is a new model choice [7, 8].
Identification $a_\sigma = \eta c^2\sqrt{\Lambda_{\rm eff}/3}$	Potentially distinctive only if derived from the membrane action and supported by joint data.
Single framework linking galaxies, expansion anisotropy and boundary photons	Conceptually distinctive, but presently speculative and dependent on several unproven closures.
Neighbouring-universe JWST interpretation	Distinctive hypothesis requiring the full optical, redshift, flux and angular tests derived here.

38 Minimum viable research sequence

1. Solve Equation (2.1) for a public set of galaxy baryonic maps using one a_σ.
2. Use the same potentials in Equations (13.1)–(13.6) and compare with galaxy-galaxy and strong lensing.
3. Specify $\mathcal{L}_{\rm stab}$, derive the full linearised mode spectrum and calculate post-Newtonian parameters.
4. Select a potential $V(q)$, integrate Equations (15.2), (17.4) and (18.1), then test the shared $a_\sigma(z)$ relation.
5. Implement the perturbation system and calculate CMB, matter power and lensing spectra. Do not infer success from the background alone.
6. Test the shear quadrupole before introducing a more flexible inhomogeneous boundary model.
7. Only after the gravitational sector survives, fit boundary optics using Equations (26.3), (27.1) and (28.2).

Bottom line. The original galaxy equation can be extended into a coherent mathematical programme, but not into a unique theory by algebra alone. The decisive task is to choose one complete covariant action, derive every displayed sector from it where possible, and fit all observations with one parameter set. The strongest near-term tests are the rotation-lensing identity and the predicted relation between the galaxy acceleration scale and the cosmological tension scale.

A Derivation of the spherical root

Starting from Equation (3.2), square both sides:

$$\frac{g^4}{g^2 + a_\sigma^2} = g_{\mathrm{N}}^2. \tag{A.1}$$

Set $z = g^2$. Then

$$z^2 - g_{\mathrm{N}}^2 z - g_{\mathrm{N}}^2 a_\sigma^2 = 0. \tag{A.2}$$

The physical root is

$$z = \frac{g_{\mathrm{N}}^2 + \sqrt{g_{\mathrm{N}}^4 + 4g_{\mathrm{N}}^2 a_\sigma^2}}{2}, \tag{A.3}$$

which gives Equation (3.3).

B Derivation of the constant deep-curvature deflection

With $\Phi = V^2 \ln \sqrt{b^2 + \ell^2}$,

$$\nabla_\perp \Phi = \frac{V^2 b}{b^2 + \ell^2}\,\hat{\boldsymbol{b}}. \tag{B.1}$$

Using

$$\int_{-\infty}^{\infty} \frac{b\,\mathrm{d}\ell}{b^2 + \ell^2} = \pi, \tag{B.2}$$

Equation (13.2) gives $\alpha = 2\pi V^2/c^2$.

C Dimensional checks

Quantity	Dimensions in SI
Φ	$\mathrm{m}^2\,\mathrm{s}^{-2}$
$\nabla\Phi$, a_σ	$\mathrm{m}\,\mathrm{s}^{-2}$
Λ_4	m^{-2}
$c^2\sqrt{\Lambda_4}$	$\mathrm{m}\,\mathrm{s}^{-2}$
$M = a_\sigma/c^2$	m^{-1}
κ_5^2 **in** $c = \hbar = 1$	length^3
σ **in** $c = \hbar = 1$	length^{-4}

D Compact equation set

The minimum coupled system proposed in this document is

$$\nabla \cdot \left[\mu_A(|\nabla\Phi|/a_\sigma)\nabla\Phi\right] = 4\pi G\rho_{\mathrm{b}}, \tag{C1}$$

$$a_\sigma = \eta c^2\sqrt{\Lambda_{\mathrm{eff}}/3}, \tag{C2}$$

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + 8\pi G T_{\mu\nu} + \kappa_5^4 \Pi_{\mu\nu} - E_{\mu\nu}, \tag{C3}$$

$$H^2 = \Lambda_4/3 + (8\pi G/3)\rho(1 + \rho/2\sigma) + \mathcal{C}/a^4 - k/a^2, \tag{C4}$$

$$\ddot{q} + 3H\dot{q} + V_{,q} = 0, \tag{C5}$$

$$\boldsymbol{\alpha} = c^{-2} \int \nabla_\perp(\Phi + \Psi)\, \mathrm{d}\ell, \tag{C6}$$

$$-k^2\Psi = 4\pi G a^2 \mu_L \rho_m \Delta_m, \quad \Phi = \gamma\Psi, \tag{C7}$$

$$\ddot{\beta}_i + 3H\dot{\beta}_i = 8\pi G \pi_i, \tag{C8}$$

$$[\mathscr{K}_{ab}] - h_{ab}[\mathscr{K}] = -(8\pi G/c^4) S^{\Sigma}_{ab}, \tag{C9}$$

$$1 + z_{\mathrm{obs}} = \frac{-u_{\mathrm{e}} \cdot k_-}{-U_\Sigma \cdot k_-} \frac{-U_\Sigma \cdot k_+}{-u_{\mathrm{o}} \cdot k_+}. \tag{C10}$$

Equations (C1)–(C10) are not all derivable from one completed action yet. Their purpose is to make every missing assumption visible.

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