Membrane Curvature Theory

Revised Candidate Mathematical Framework

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Abstract

This document reformulates the Dark Matter Energy Theory (DMET) around its intended central principle: mass–energy produces spacetime curvature, and gravity is the common physical origin to be tested across local, galactic and cosmological scales. The revised framework keeps a nonlinear AQUAL-type weak-field equation as a provisional galactic phenomenology; embeds possible geometric corrections in effective four- and five-dimensional equations; replaces the informal idea of neighbouring universes "pulling our universe outward" with the physically sharper language of curvature gradients, geodesic deviation, tidal fields and membrane strain; and treats the cross-region/JWST interpretation as a separate high-speculation optical extension. The framework is not a unique derivation of DMET and is not a validated replacement for General Relativity or ΛCDM. Its purpose is to expose the assumptions required for a falsifiable theory.

Contents

1 Reading guide and status labels

Label	Meaning
Established	Standard equation or formalism from established GR, brane gravity, lensing or modified-gravity literature.
Core	A mathematical statement of the intended DMET physical idea.
Ansatz	A modelling choice that is not derived uniquely from the conceptual website.
Extension	A speculative addition, especially neighbouring-region and photon-transfer ideas.
Failure criterion	A result that would reject or force substantial revision of the displayed implementation.

Metric signature is $(-,+,+,+)$. Greek indices are four-dimensional; capital Latin indices are five-dimensional. Factors of c are displayed when useful.

2 Central physical statement

Core. DMET begins with the standard relation "mass–energy produces curvature" and asks whether a larger connected geometry changes the effective gravitational response at galactic and inter-region scales. Dark matter and dark energy are therefore not postulated as two new substances at the outset.

The standard Einstein equation is

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{1}$$

A generic way to represent additional geometric information perceived by a four-dimensional observer is

$$G_{\mu\nu} + \mathcal{M}_{\mu\nu} = \frac{8\pi G}{c^4} T^{(b)}_{\mu\nu}, \tag{2}$$

where $\mathcal{M}_{\mu\nu}$ is not yet specified. A completed DMET would have to derive $\mathcal{M}_{\mu\nu}$ from the larger membrane/bulk geometry, not insert it independently for each phenomenon.

Equation (2) defines an effective geometric stress tensor

$$T^{(\mathrm{geom})}_{\mu\nu} \equiv -\frac{c^4}{8\pi G}\mathcal{M}_{\mu\nu}. \tag{3}$$

Calling this object "geometric dark matter" or "geometric dark energy" is an interpretation; its observable content comes only from equations of motion, boundary conditions and data.

Part I
Galactic weak-field sector

3 Provisional nonlinear curvature equation

Ansatz. The following equation is an explicit AQUAL-family implementation. It is retained because it turns the qualitative "extended curvature" statement into a falsifiable galaxy model, not because it has yet been derived from the membrane geometry.

Let Φ be the gravitational potential and ρ_b the baryonic density:

$$\nabla \cdot \left[\mu_A\left(\frac{|\nabla\Phi|}{a_M}\right)\nabla\Phi\right] = 4\pi G\rho_b, \tag{4}$$

with

$$\mu_A(x) = \frac{x}{\sqrt{1+x^2}}. \tag{5}$$

The limits are $\mu_A \to 1$ for $x \gg 1$ and $\mu_A \to x$ for $x \ll 1$.

The non-relativistic action

$$S_\Phi = -\int dt\, d^3x \left[\frac{a_M^2}{8\pi G}F\left(\frac{|\nabla\Phi|^2}{a_M^2}\right) + \rho_b\Phi\right] \tag{6}$$

with

$$F(y) = \sqrt{y(1+y)} - \operatorname{arsinh}\sqrt{y} \tag{7}$$

produces Eq. (4) on variation.

4 Spherical solution and direct prediction

For $g = |\nabla\Phi|$ and $g_N = GM_b(r)/r^2$,

$$\frac{g^2}{\sqrt{g^2 + a_M^2}} = g_N, \tag{8}$$

so

$$g = \left[\frac{g_N^2 + \sqrt{g_N^4 + 4a_M^2 g_N^2}}{2}\right]^{1/2}. \tag{9}$$

At $g_N \ll a_M$,

$$g \simeq \frac{\sqrt{GM_b a_M}}{r}, \qquad \Phi \simeq \sqrt{GM_b a_M}\ln\frac{r}{r_0}, \tag{10}$$

and

$$\boxed{v_\infty^4 = GM_b a_M.} \tag{11}$$

A universal a_M is therefore a non-negotiable prediction of this displayed version.

5 Effective curvature density

Rewrite the potential as

$$\nabla^2\Phi = 4\pi G(\rho_b + \rho_{\rm curv}), \tag{12}$$

with

$$\rho_{\rm curv} = \frac{1}{4\pi G}\nabla\cdot[(1-\mu_A)\nabla\Phi]. \tag{13}$$

This is an effective geometric reconstruction, not a declaration that an actual material density exists.

6 Central black holes and galaxy sources

The source term is the complete baryonic mass distribution. Central supermassive black holes contribute to ρ_b but are not assumed to dominate the outer field. A DMET derivation must explain why the combined source creates the required weak-field response while preserving local tests of GR.

7 External-field effect of the nonlinear equation

Because Eq. (4) is nonlinear, a constant external field can change the internal solution. For $\nabla\Phi = \boldsymbol{g}_e + \nabla\varphi$ with $|\nabla\varphi| \ll g_e$,

$$\partial_i\left[\mu_e(\delta_{ij} + L_e\hat{e}_i\hat{e}_j)\partial_j\varphi\right] = 4\pi G\rho_{\rm int}, \tag{14}$$

where

$$\mu_e = \mu_A(g_e/a_M), \qquad L_e = \left.\frac{d\ln\mu_A}{d\ln x}\right|_{g_e/a_M} = \frac{1}{1+(g_e/a_M)^2}. \tag{15}$$

This is a direct consequence of the chosen AQUAL equation and supplies an observational test in satellites and other low-acceleration systems.

Part II
Relativistic and higher-dimensional geometry

8 Five-dimensional brane framework

Established. This section uses standard brane-world projection mathematics. Its presence shows one way a larger geometry can influence four-dimensional gravity; it is not a derivation of the DMET website.

Let the brane be embedded in a five-dimensional bulk with induced metric

$$g_{\mu\nu} = G_{AB}e^A_\mu e^B_\nu, \qquad e^A_\mu = \frac{\partial X^A}{\partial x^\mu}. \tag{16}$$

A minimal action is

$$S = \frac{1}{2\kappa_5^2} \int_{\mathcal{M}} d^5 X \sqrt{-G}(R_5 - 2\Lambda_5) + \int_{\mathcal{B}} d^4 x \sqrt{-g}[-\sigma + \mathcal{L}_b + \mathcal{L}_{\rm mem}], \tag{17}$$

with the appropriate boundary term understood.

For Z_2 symmetry, the Israel junction condition gives

$$[K_{\mu\nu}] = -\kappa_5^2 \left(S_{\mu\nu} - \frac{1}{3} S g_{\mu\nu} \right). \tag{18}$$

Projecting the five-dimensional field equations onto the brane yields

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + 8\pi G T_{\mu\nu} + \kappa_5^4 \Pi_{\mu\nu} - E_{\mu\nu}. \tag{19}$$

The projected Weyl tensor $E_{\mu\nu}$ carries nonlocal bulk gravitational information. Equation (19) is not closed from brane data alone.

Closure requirement. A successful DMET cannot simply rename $E_{\mu\nu}$ "extended curvature". It must calculate it from a bulk solution, boundary data or a well-defined covariant closure.

9 A candidate covariant closure

One possible effective four-dimensional closure uses a unit timelike vector U^μ and acceleration invariant

$$U^\mu U_\mu = -1, \qquad a_\mu = U^\nu \nabla_\nu U_\mu, \qquad Y = \frac{a_\mu a^\mu}{M^2}, \qquad M = \frac{a_M}{c^2}. \tag{20}$$

A candidate action is

$$S_4 = \frac{c^4}{16\pi G} \int d^4 x \sqrt{-g} \left[R - 2\Lambda_{\rm eff} - 2M^2 \mathcal{G}(Y) + \lambda(U^2 + 1) + \mathcal{L}_{\rm stab} \right] + S_b, \tag{21}$$

where $\mathcal{G}(Y) = F(Y) - Y$. The unspecified $\mathcal{L}_{\rm stab}$ is essential: ghosts, gradients, mode speeds and preferred-frame effects must be derived before this can be considered viable.

10 Lensing consistency

Write

$$ds^2 = -\left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 + \left(1 - \frac{2\Psi}{c^2}\right) d\boldsymbol{x}^2. \tag{22}$$

Slow matter responds mainly to Φ, while light responds to $\Phi + \Psi$. The weak deflection is

$$\boldsymbol{\alpha} = \frac{1}{c^2} \int \nabla_\perp (\Phi + \Psi)\, d\ell. \tag{23}$$

For no slip, $\Psi \simeq \Phi$, the deep logarithmic potential gives

$$\alpha \simeq \frac{2\pi}{c^2} \sqrt{G M_b a_M} \tag{24}$$

until environmental truncation becomes important.

Failure criterion. If the same relativistic geometry that fits galaxy dynamics cannot reproduce galaxy and cluster lensing without a separately tuned halo, the core displayed implementation fails.

11 Merging clusters and dynamical curvature

An equilibrium elliptic equation tracks the present baryonic distribution and therefore does not automatically produce lensing peaks displaced from collisional gas. A possible dynamical Weyl/curvature sector could schematically obey

$$\tau_W^2 (\ddot{\rho}_W + 3H\dot{\rho}_W) - \ell_W^2 \nabla^2 \rho_W + \rho_W = \rho_{\rm curv}^{\rm eq}[\rho_b]. \tag{25}$$

This is an optional extension, not a rescue that may be tuned independently for each cluster. A real bulk theory must determine τ_W and ℓ_W.

Part III

Large-scale curvature, tides and expansion

12 Why "external pull" is not enough

Core. The revised DMET expansion idea is based on curvature gradients and relative acceleration. A uniform acceleration of the entire observable region is locally unobservable and does not constitute cosmic expansion.

A Newtonian analogy for neighbouring masses is

$$\boldsymbol{g}_{\text{ext}}(\boldsymbol{x}) = -G\sum_i M_i \frac{\boldsymbol{x}-\boldsymbol{X}_i}{|\boldsymbol{x}-\boldsymbol{X}_i|^3}. \tag{26}$$

The relevant local quantity is its gradient,

$$\mathcal{T}_{ij} = \partial_j g_{\text{ext},i}, \qquad \Delta a_i \simeq \mathcal{T}_{ij}\xi^j. \tag{27}$$

In GR the invariant statement is geodesic deviation,

$$\frac{D^2\xi^\mu}{D\tau^2} = -R^\mu{}_{\alpha\nu\beta}u^\alpha\xi^\nu u^\beta. \tag{28}$$

Equation (28) is the natural language for asking whether curvature sourced outside our observed region can alter relative separations inside it.

13 Raychaudhuri constraint

For a timelike congruence with expansion θ, shear $\sigma_{\mu\nu}$, vorticity $\omega_{\mu\nu}$ and four-acceleration a^μ,

$$\frac{d\theta}{d\tau} = -\frac{\theta^2}{3} - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^\mu u^\nu + \nabla_\mu a^\mu. \tag{29}$$

For geodesic, irrotational matter, ordinary positive-density matter focuses worldlines. Therefore an accelerating nearly homogeneous universe is not obtained merely by adding more ordinary attractive masses outside the region. DMET must show how the larger geometry changes the effective $R_{\mu\nu}u^\mu u^\nu$, generates an effective negative-pressure sector, or produces an inhomogeneous distance–redshift relation that matches observations.

14 Effective geometric equation of state

For an effective homogeneous geometric sector,

$$p_g = w_g\rho_g c^2, \tag{30}$$

and the acceleration equation is

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho_m + \rho_g(1+3w_g)\right]. \tag{31}$$

Late-time acceleration requires

$$\boxed{w_g < -\frac{1}{3}.} \tag{32}$$

Thus "neighbouring curvature" becomes a dark-energy explanation only if the projected geometry generates such an effective stress or an observationally equivalent non-FLRW effect.

15 Dynamical strain as one candidate mechanism

Keep the fundamental brane tension σ fixed and introduce a strain scalar q,

$$S_q = -\int d^4x\sqrt{-g}\left[\frac{1}{2}\nabla_\mu q\nabla^\mu q + V(q)\right]. \tag{33}$$

For homogeneous q,

$$\rho_q = \frac{1}{2}\dot{q}^2 + V(q), \qquad p_q = \frac{1}{2}\dot{q}^2 - V(q), \qquad \ddot{q} + 3H\dot{q} + V_{,q} = 0. \tag{34}$$

Potential domination gives $w_q \simeq -1$. In DMET language this can be interpreted as a membrane strain/tension response, but the interpretation becomes meaningful only if q is derived from the larger geometry rather than added ad hoc.

16 Background expansion

A standard brane Friedmann equation is

$$H^2 = \frac{\Lambda_4}{3} + \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\sigma}\right) + \frac{C}{a^4} - \frac{k}{a^2}. \tag{35}$$

With an added strain sector ρ_q it becomes, schematically,

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_q) + \text{brane corrections}. \tag{36}$$

A credible model must reproduce supernova distances, BAO, CMB distance scales and growth with one parameter set.

17 Directional expansion

For Bianchi-I,

$$ds^2 = -dt^2 + a^2(t)\sum_i e^{2\beta_i(t)}(dx^i)^2, \qquad \sum_i \beta_i = 0, \tag{37}$$

with directional rates

$$H_i = H + \dot{\beta}_i. \tag{38}$$

Anisotropic stress π_i sources shear,

$$\ddot{\beta}_i + 3H\dot{\beta}_i = 8\pi G\pi_i. \tag{39}$$

For a source seen along n^i,

$$H(\boldsymbol{n}) = H + \sum_i n_i^2\dot{\beta}_i. \tag{40}$$

A homogeneous strain therefore predicts a quadrupolar directional signature. Any claimed directional expansion must respect existing limits on isotropy.

Part IV
Early-universe and structure requirements

18 Linear perturbations

Use conformal Newtonian gauge,

$$ds^2 = a^2(\tau)[-(1+2\Psi)d\tau^2 + (1-2\Phi)d\boldsymbol{x}^2]. \tag{41}$$

A phenomenological modified-gravity description is

$$-k^2\Psi = 4\pi G a^2 \mu_L(k,a)\rho_m\Delta_m, \tag{42}$$
$$\Phi = \gamma(k,a)\Psi, \tag{43}$$
$$-k^2(\Phi+\Psi) = 8\pi G a^2 \Sigma(k,a)\rho_m\Delta_m. \tag{44}$$

These linear functions are not the same object as the nonlinear galaxy interpolation μ_A.

A no-particle dark-matter model must provide a geometric perturbation mode that behaves approximately like a pressureless clustering component before recombination,

$$w_{\rm geom} \simeq 0, \qquad c^2_{s,\rm geom} \ll 1. \tag{45}$$

Without such a mode, the model will not reproduce the gravitational potentials needed for the observed CMB acoustic structure and subsequent structure growth.

Part V
Boundary optics and the JWST extension

19 Two distinct claims: lensing versus cross-region origin

Extension. A galaxy can be gravitationally lensed without coming from another universe-region. Conversely, a cross-region source needs a photon-transfer/redshift mechanism in addition to any lensing. These ideas must not be conflated.

For an ordinary thin lens, the source and image angles satisfy

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta}), \tag{46}$$

with Jacobian

$$\mathsf{A}_{ij} = \frac{\partial \beta_i}{\partial \theta_j}, \qquad \mu_{\rm lens} = \frac{1}{|\det \mathsf{A}|}. \tag{47}$$

Lensing can make an object appear brighter and alter its apparent angular size or multiplicity. In a stationary spacetime it does not manufacture an arbitrary common spectral redshift.

20 Geometric boundary matching

Model a boundary hypersurface Σ with induced metric h_{ab}. The thin-wall matching condition is

$$[K_{ab}] - h_{ab}[K] = -\frac{8\pi G}{c^4} S^{\Sigma}_{ab}. \tag{48}$$

A soft boundary requires solving through a finite-thickness stress profile.

For electromagnetic propagation with kinetic coefficient Z,

$$\nabla_\mu (Z F^{\mu\nu}) = J^\nu, \tag{49}$$

with tangential field matching and appropriate normal-flux conditions. A transparent boundary is an additional physical requirement, not automatic.

21 Cross-region redshift

In geometric optics,

$$k_\mu = \nabla_\mu \Theta, \qquad k_\mu k^\mu = 0, \qquad k^\nu \nabla_\nu k^\mu = 0, \tag{50}$$

and an observer measures

$$\omega = -u_\mu k^\mu. \tag{51}$$

If a photon crosses an interface with four-velocity U^μ_Σ, the total redshift must be computed from the complete path. A schematic matching expression is

$$1 + z_{\rm obs} = \frac{-u_e \cdot k^-}{-U_\Sigma \cdot k^-} \, \frac{-U_\Sigma \cdot k^+}{-u_o \cdot k^+}. \tag{52}$$

For a non-dispersive boundary all identified spectral features must share one redshift. Time intervals must exhibit the corresponding time-dilation factor.

22 Flux and magnification

For source luminosity L_e, boundary transmission $\mathcal{T}$ and geometric magnification $\mu_{\rm geom}$,

$$F_o = \frac{\mathcal{T}\, \mu_{\rm geom} L_e}{4\pi d_L^2}. \tag{53}$$

Passive transmission alone has $\mathcal{T} \leq 1$ and cannot make a source intrinsically more luminous. Apparent over-luminosity requires magnification and/or an altered distance–redshift mapping. This is a decisive constraint on the JWST extension.

23 Separate-brane photon portal

If neighbouring regions are truly separate branes, ordinary photons do not automatically cross. A minimal two-state toy model is

$$i\frac{d}{d\lambda}\begin{pmatrix}A_+\\A_-\end{pmatrix}=\begin{pmatrix}\Delta_+ & \epsilon\\ \epsilon & \Delta_-\end{pmatrix}\begin{pmatrix}A_+\\A_-\end{pmatrix}, \tag{54}$$

with conversion probability

$$P_{-\to+}=\frac{4\epsilon^2}{\Delta^2+4\epsilon^2}\sin^2\left[\frac{L}{2}\sqrt{\Delta^2+4\epsilon^2}\right]. \tag{55}$$

This is an optional portal model and would introduce additional spectral, polarization or leakage constraints.

Part VI
Consistency, computation and falsification

24 Conservation

A closed effective four-dimensional theory must satisfy

$$\nabla_\mu\left(T_b^{\mu\nu}+T_{\text{geom}}^{\mu\nu}\right)=0. \tag{56}$$

In a brane description, apparent exchange with the bulk is represented by a nonzero transfer vector Q^ν. Any varying tension, photon leakage or boundary exchange must appear in the energy–momentum budget.

25 Stability checklist

Any covariant completion must demonstrate:

1. positive kinetic energies for scalar, vector and tensor modes;
2. non-negative squared propagation speeds;
3. tensor waves propagating at the observed luminal speed;
4. acceptable post-Newtonian and preferred-frame parameters;
5. absence of generic caustics or gradient singularities;
6. a well-posed initial-value problem.

26 Joint parameter discipline

Let d contain galaxy velocities, lensing, expansion, growth and anisotropy observables, with model $m(\theta)$ and covariance C:

$$-2\ln\mathcal{L}=(d-m)^TC^{-1}(d-m)+\ln\det C+\text{const}. \tag{57}$$

One shared parameter vector must be used. Optional cluster-memory, strain and photon-portal parameters cannot be fitted independently system by system without destroying falsifiability.

27 Revised hierarchy of claims

Layer	Claim
DMET-Core	Larger geometry modifies the effective weak-field curvature around baryonic structures and must jointly predict dynamics and lensing.
DMET-Expansion	External/bulk curvature gradients generate tidal/strain geometry whose effective stress or inhomogeneous dynamics reproduces cosmic acceleration.
DMET-Boundary	Photons from another universe-region can reach us through a calculable interface or portal and may produce a distinctive high-redshift/lensing population.

Failure of the boundary layer need not imply failure of the core galaxy hypothesis. Failure of the shared gravitational geometry to fit dynamics and lensing would strike much closer to the core.

28 Decisive failure criteria

The displayed framework should be rejected or substantially revised if:

1. no universal a_M fits high-quality isolated galaxy rotation data within baryonic uncertainties;
2. a relativistic completion that fits dynamics gives the wrong lensing amplitude or morphology;
3. cluster offsets require arbitrary system-specific memory parameters;
4. no stable pressureless geometric mode reproduces CMB potentials and structure growth;
5. the large-scale curvature/strain sector cannot reproduce $H(z)$ while respecting isotropy and conservation;
6. the theory requires ghosts, pathological gradients, non-luminal tensor waves or unacceptable preferred-frame effects;
7. a cross-region source model cannot reproduce coherent spectral redshift, flux, image morphology, time dilation and sky correlations;
8. the photon transfer required for JWST candidates conflicts with electromagnetic constraints elsewhere.

29 Minimum viable research sequence

1. Solve Eq. (4) for public baryonic galaxy maps with one a_M.
2. Use the same relativistic potentials to predict lensing.
3. Test merging clusters and local external-field effects.
4. Specify one complete covariant closure and derive its perturbation spectrum and local-gravity limits.
5. Derive the cosmological geometric/strain sector and fit expansion plus growth.
6. Compute CMB and matter-power observables.
7. Test directional/shear signatures.
8. Only then fit any boundary/JWST population.

30 Compact equation set

A concise summary of the revised programme is

$$\nabla \cdot [\mu_A(|\nabla\Phi|/a_M)\nabla\Phi] = 4\pi G\rho_b, \tag{58}$$

$$G_{\mu\nu} + \mathcal{M}_{\mu\nu} = \frac{8\pi G}{c^4} T^{(b)}_{\mu\nu}, \tag{59}$$

$$\frac{D^2\xi^\mu}{D\tau^2} = -R^\mu{}_{\alpha\nu\beta} u^\alpha \xi^\nu u^\beta, \tag{60}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}[\rho_m + \rho_g(1+3w_g)], \tag{61}$$

$$\boldsymbol{\alpha} = c^{-2} \int \nabla_\perp(\Phi+\Psi)d\ell, \tag{62}$$

$$H(\boldsymbol{n}) = H + \sum_i n_i^2 \dot{\beta}_i, \tag{63}$$

$$\omega = -u_\mu k^\mu, \tag{64}$$

$$\mu_{\rm lens} = |\det \mathsf{A}|^{-1}. \tag{65}$$

These equations are deliberately not presented as if they already arise from one completed action. The research task is to reduce them to a single self-consistent geometry.

31 Bottom line

The central DMET idea is strongest when stated conservatively: gravity is not being replaced; rather, the theory asks whether the geometry responsible for gravity has been incompletely modelled when the universe is embedded in a larger connected structure. The short-term scientific test is not the existence of neighbouring universes. It is whether one geometric law can predict galaxy motion and lensing from baryonic sources, survive local and cluster tests, and admit a stable relativistic extension. Only after that should the inter-region expansion and JWST optics be treated as quantitative predictions.

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