Membrane Curvature Model

A First Mathematical Formulation

Inspired by the conceptual model presented at
darkmatterenergytheory.com

Purpose. This note turns the website's verbal idea into one explicit, testable equation. It is an exploratory mathematical realisation, not a derivation uniquely implied by the website and not a validated physical theory.

Status: Speculative research formulation
Scope: Non-relativistic galactic dynamics and a proposed tension scale
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Executive summary

The website proposes that ordinary matter creates curvature basins in a cosmic membrane, while membrane tension controls how broadly those basins extend. A concise way to express that proposal is to modify the non-relativistic Poisson equation so that gravity becomes nonlinear below an acceleration scale set by the membrane tension.

$$\boxed{\nabla\cdot\left[\frac{|\nabla\Phi|}{\sqrt{|\nabla\Phi|^2+a_\sigma^2}}\nabla\Phi\right]=4\pi G\rho_{\mathrm{b}},\qquad a_\sigma=\eta c^2\sqrt{\frac{\Lambda_\sigma}{3}}} \tag{1}$$

For an isolated spherical galaxy, this equation predicts an asymptotically flat rotation curve and the baryonic scaling relation

$$\boxed{v_\infty^4=GM_{\mathrm{b}}a_\sigma.} \tag{2}$$

Its key proposed novelty is not the nonlinear gravity structure, which belongs to the established AQUAL/MOND family, but the identification

$$\boxed{a_0=a_\sigma=\eta c^2\sqrt{\frac{\Lambda_\sigma}{3}}=\eta cH_\sigma.} \tag{3}$$

This ties the acceleration scale observed in galaxy dynamics to a cosmological scale associated with membrane tension.

Scientific status. Equation (1) is a plausible mathematical implementation of the concept, not a consequence derived from a complete membrane theory. A relativistic completion is still required for gravitational lensing, cosmology, structure formation and photon propagation between neighbouring regions.

1 Definitions and assumptions

The formulation uses the following quantities:

Symbol	Meaning
Φ	Gravitational potential, interpreted here as the local depth of a membrane-curvature basin.
ρ_{b}	Observed baryonic mass density, including stars, gas and other ordinary matter.
G	Newton's gravitational constant.
c	Speed of light.
Λ_σ	Effective cosmological curvature attributed to membrane tension.
a_σ	Acceleration scale below which the extended-curvature response becomes important.
η	Universal dimensionless coupling between the tension-driven cosmological scale and galactic dynamics.

The central assumptions are:

1. Baryonic matter is the direct source on the right-hand side of the field equation.
2. The membrane response becomes nonlinear when $|\nabla\Phi|$ is comparable to or smaller than a_σ.
3. The same membrane tension that contributes to cosmic acceleration sets a_σ through Equation (3).
4. The coupling η is universal. It must not be adjusted independently for each galaxy.

2 The nonlinear field equation

In ordinary Newtonian gravity,

$$\nabla^2\Phi = 4\pi G\rho_{\rm b}. \tag{4}$$

The proposed membrane response replaces the constant gravitational susceptibility by

$$\mu\left(\frac{|\nabla\Phi|}{a_\sigma}\right) = \frac{|\nabla\Phi|}{\sqrt{|\nabla\Phi|^2 + a_\sigma^2}}. \tag{5}$$

Equation (1) can therefore be written compactly as

$$\nabla\cdot\left[\mu\left(\frac{|\nabla\Phi|}{a_\sigma}\right)\nabla\Phi\right] = 4\pi G\rho_{\rm b}. \tag{6}$$

The interpolation function has the required limits:

$$\mu \longrightarrow 1, \qquad\qquad |\nabla\Phi| \gg a_\sigma, \tag{7}$$

$$\mu \longrightarrow \frac{|\nabla\Phi|}{a_\sigma}, \qquad\qquad |\nabla\Phi| \ll a_\sigma. \tag{8}$$

Thus the model recovers Newtonian gravity in strong fields and produces an extended response in weak fields.

3 Spherical solution and rotation curves

For a spherically symmetric system, define

$$g(r) = |\nabla\Phi|, \qquad g_{\rm N}(r) = \frac{GM_{\rm b}(r)}{r^2}. \tag{9}$$

Integrating Equation (1) over a sphere gives

$$\frac{g^2}{\sqrt{g^2 + a_\sigma^2}} = g_{\rm N}. \tag{10}$$

Solving for the positive physical root gives

$$\boxed{g(r) = \left[\frac{g_{\rm N}^2 + \sqrt{g_{\rm N}^4 + 4a_\sigma^2 g_{\rm N}^2}}{2}\right]^{1/2}.} \tag{11}$$

The circular speed is then

$$v_{\rm c}^2(r) = r\,g(r). \tag{12}$$

3.1 Strong-curvature limit

When $g_{\rm N} \gg a_\sigma$,

$$g \simeq g_{\rm N}, \tag{13}$$

so standard Newtonian dynamics is recovered to leading order.

3.2 Extended-curvature limit

When $g_{\rm N} \ll a_\sigma$,

$$g \simeq \sqrt{g_{\rm N} a_\sigma} = \frac{\sqrt{GM_{\rm b} a_\sigma}}{r}. \tag{14}$$

The far-field potential is therefore logarithmic:

$$\Phi(r) \simeq \sqrt{GM_{\rm b} a_\sigma}\,\ln\!\left(\frac{r}{r_0}\right), \tag{15}$$

where r_0 is an arbitrary reference radius. Since $g \propto 1/r$, the orbital speed approaches a constant and Equation (2) follows.

Immediate falsifiable prediction. For sufficiently isolated galaxies, the quantity

$$\frac{v_\infty^4}{GM_{\rm b}}$$

should approach one universal value, a_σ. The model does not permit a separate, freely fitted dark-halo mass for each galaxy.

4 Connecting galactic dynamics to membrane tension

Define the expansion rate associated with the tension component as

$$H_\sigma = c\sqrt{\frac{\Lambda_\sigma}{3}}. \tag{16}$$

The acceleration scale is then

$$a_\sigma = \eta c H_\sigma. \tag{17}$$

This is the concise mathematical statement of the proposed unification:

> The same membrane tension that contributes to accelerated expansion sets the acceleration threshold below which gravity develops an extended curvature basin.

In a conventional five-dimensional brane construction, an illustrative effective four-dimensional cosmological curvature is

$$\Lambda_\sigma = \frac{\kappa_5^2}{2}\left(\Lambda_5 + \frac{\kappa_5^2\sigma^2}{6}\right), \qquad (c = 1), \tag{18}$$

where σ is brane tension, Λ_5 is bulk curvature and κ_5 is the five-dimensional gravitational coupling. Equation (18) is borrowed from established brane-world projection mathematics. It is illustrative and model-dependent, rather than derived from the website's conceptual description.

5 Action formulation

Equation (1) can be obtained by varying the non-relativistic action

$$S_\Phi = -\int \mathrm{d}t\ \mathrm{d}^3x \left[\frac{a_\sigma^2}{8\pi G} F\left(\frac{|\nabla\Phi|^2}{a_\sigma^2}\right) + \rho_\mathrm{b}\Phi\right], \tag{19}$$

with

$$F(y) = \sqrt{y(1+y)} - \operatorname{arsinh}\sqrt{y}. \tag{20}$$

Since

$$F'(y) = \sqrt{\frac{y}{1+y}}, \tag{21}$$

variation of S_Φ with respect to Φ yields Equation (1). An action is important because it supplies a consistent variational structure and a starting point for deriving conservation laws. It does not, by itself, provide the required relativistic theory.

6 Originality and relationship to existing work

The nonlinear Poisson equation used here is an explicit member of the AQUAL/MOND class introduced by Bekenstein and Milgrom in 1984. The logarithmic far-field potential and the relation $v_\infty^4 = GMa_0$ are therefore not new results.

The potentially original scientific claim would be the derivation of

$$a_0 = \eta c^2\sqrt{\frac{\Lambda_\sigma}{3}} \tag{22}$$

from a specified membrane geometry, together with predictions that distinguish this mechanism from existing modified-gravity and brane-world models.

Projected bulk curvature has also been investigated previously as a geometric explanation for flat galactic rotation curves. Any new model must identify exactly where its field content, boundary conditions or observable predictions differ from that literature.

7 What the equation does not yet explain

Equation (1) is sufficient for an initial rotation-curve test, but it does not yet provide:

- a relativistic spacetime metric and a unique prediction for gravitational lensing;
- a modified Friedmann equation for the full expansion history $H(z)$;
- cosmic microwave background anisotropies or large-scale structure growth;
- the dynamics of clusters and merging systems;
- directional expansion generated by membrane boundaries;
- photon propagation across a boundary between neighbouring universes;
- a stability analysis, causal structure or quantum completion.

A credible next stage would define a covariant action for the four-dimensional membrane and its higher-dimensional bulk, derive the weak-field equation as a controlled limit, and fit one universal parameter set to rotation curves, lensing, cosmology and structure formation.

8 Minimum empirical programme

The model can be developed in a disciplined sequence:

1. **Galaxy test.** Fit Equation (11) to rotation-curve data using observed baryonic mass distributions and one universal a_σ.
2. **External-field test.** Calculate whether nearby masses alter the nonlinear solution and compare with satellite and wide-binary systems.
3. **Lensing test.** Construct a relativistic metric and verify that the same parameters reproduce lensing without an independently chosen halo.
4. **Cosmological test.** Derive $H(z)$ from the tension sector and compare it with supernova, BAO and cosmic microwave background constraints.
5. **Distinctive prediction.** State at least one numerical result that differs from both standard ΛCDM and existing MOND or brane-world formulations.

Bottom line. This document supplies the requested first equation and its main consequences. The decisive scientific task is now to derive the equation from a fully specified membrane theory rather than choosing it because it produces the desired galaxy phenomenology.

References

[1] Dark Matter Energy Theory, conceptual model and theory pages, darkmatterenergytheory.com.

[2] J. D. Bekenstein and M. Milgrom, "Does the missing mass problem signal the breakdown of Newtonian gravity?", *The Astrophysical Journal*, 286, 7–14 (1984), DOI: 10.1086/162570.

[3] R. Maartens and K. Koyama, "Brane-World Gravity", *Living Reviews in Relativity*, 13, 5 (2010), arXiv:1004.3962.

[4] T. Harko and K. S. Cheng, "Galactic metric, dark radiation, dark pressure and gravitational lensing in brane world models" (2005), arXiv:astro-ph/0509576.