Membrane Curvature Model

A Revised First Mathematical Formulation

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Abstract

The Dark Matter Energy Theory (DMET) begins from one organising idea: mass–energy curves spacetime, and gravity is the observable consequence of that geometry. Rather than treating dark matter and dark energy as two unrelated unknown substances, DMET asks whether the gravitational geometry already associated with mass can have additional large-scale consequences. This note gives a disciplined mathematical version of that idea. It retains an AQUAL-type weak-field equation as a provisional description of galaxy dynamics, introduces a tidal-curvature language for the proposed influence of neighbouring massive universe-regions, and separates the highly speculative cross-region/JWST hypothesis from the core gravitational model. None of these equations is claimed to be a completed or validated theory.

Status. The nonlinear galaxy equation below belongs to the established AQUAL/MOND mathematical family. Its use in DMET is a modelling choice, not an original derivation. The potentially distinctive DMET claim would be to derive a corresponding weak-field law and the large-scale tidal/strain sector from one covariant membrane geometry.

1 The organising principle

General Relativity relates spacetime geometry to stress–energy through

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{1}$$

DMET does not challenge the statement that mass–energy produces curvature. Its speculative step is to ask whether the effective geometry seen inside our observable universe is influenced by a larger connected geometry. Schematically,

$$\boxed{\text{mass–energy} \to \text{curvature} \to \text{gravity} \to \text{multi-scale observables}}$$

local motion and accretion | galactic binding and lensing | large-scale tidal/strain effects

The first two arrows are standard physics. The final scale-extension is the DMET hypothesis.

2 A provisional galaxy equation

Let Φ be the weak-field potential and ρ_b the observed baryonic density. A calculable toy implementation is

$$\nabla \cdot \left[\mu\left(\frac{|\nabla \Phi|}{a_M} \right) \nabla \Phi \right] = 4\pi G \rho_b, \tag{2}$$

with

$$\mu(x) = \frac{x}{\sqrt{1+x^2}}. \tag{3}$$

Here a_M is a universal acceleration scale. The required limits are

$$\mu(x) \to 1 \quad (x \gg 1), \qquad \mu(x) \to x \quad (x \ll 1). \tag{4}$$

Thus ordinary Newtonian gravity is recovered in the high-acceleration limit, while the response becomes nonlinear in the weak-field regime.

For spherical baryonic mass $M_b(r)$,

$$g_N(r) = \frac{GM_b(r)}{r^2}, \qquad g = |\nabla\Phi|, \tag{5}$$

and Eq. (2) gives

$$\frac{g^2}{\sqrt{g^2 + a_M^2}} = g_N. \tag{6}$$

The positive solution is

$$g = \left[\frac{g_N^2 + \sqrt{g_N^4 + 4a_M^2 g_N^2}}{2}\right]^{1/2}. \tag{7}$$

In the weak-field limit,

$$g \simeq \sqrt{g_N a_M} = \frac{\sqrt{GM_b a_M}}{r}, \tag{8}$$

so circular motion obeys

$$\boxed{v_\infty^4 = GM_b a_M.} \tag{9}$$

This is the baryonic Tully–Fisher scaling already associated with MOND/AQUAL. In DMET it is best viewed as a target phenomenology: a successful membrane theory should derive, rather than merely choose, the required low-acceleration response.

3 Geometric "dark matter" as an effective density

The same potential can be rewritten as an ordinary Poisson equation,

$$\nabla^2\Phi = 4\pi G(\rho_b + \rho_{\rm geom}), \tag{10}$$

with

$$\rho_{\rm geom} = \frac{1}{4\pi G}\nabla\cdot[(1-\mu)\nabla\Phi]\,. \tag{11}$$

This $\rho_{\rm geom}$ is not asserted to be a material particle density. It is the density an observer using the standard Poisson equation would infer from the additional curvature. This is the cleanest mathematical expression of the DMET dark-matter idea.

4 Why a central black hole is not enough

The central supermassive black hole contributes to the galaxy's potential, but DMET does not require it to dominate the outer rotation curve. The source ρ_b is the complete baryonic distribution: stars, gas, compact objects and the central black hole. The hypothesis concerns the *response of the larger geometry to the combined source*, not an unusually strong gravitational field from the black hole alone.

5 Neighbouring regions and the tidal requirement

Suppose, as a speculative extension, that other massive universe-regions exist in a larger connected geometry. A Newtonian analogy for their external field is

$$\boldsymbol{g}_{\rm ext}(\boldsymbol{x}) = -G\sum_i M_i \frac{\boldsymbol{x}-\boldsymbol{X}_i}{|\boldsymbol{x}-\boldsymbol{X}_i|^3}. \tag{12}$$

Equation (12) is only an analogy; a relativistic theory must replace it. Crucially, a uniform external acceleration does not create internal expansion. What changes relative separations is the tidal tensor

$$\mathcal{T}_{ij} = \partial_j g_{{\rm ext},i}, \tag{13}$$

and for two nearby worldlines separated by ξ^j,

$$\Delta a_i \simeq \mathcal{T}_{ij}\xi^j. \tag{14}$$

The relativistic form is geodesic deviation,

$$\frac{D^2\xi^\mu}{D\tau^2} = -R^\mu{}_{\alpha\nu\beta}u^\alpha \xi^\nu u^\beta. \tag{15}$$

This is a much sharper statement of the DMET large-scale idea: neighbouring mass would matter through curvature gradients and geodesic deviation, not through a simple "outward pull".

6 From external curvature to an effective expansion sector

A four-dimensional observer may parameterise additional membrane/bulk geometry as

$$G_{\mu\nu} + \mathcal{M}_{\mu\nu} = \frac{8\pi G}{c^4}T^{(b)}_{\mu\nu}, \tag{16}$$

where $\mathcal{M}_{\mu\nu}$ represents the geometric correction that must ultimately be derived from the larger theory. Moving it to the right-hand side defines an effective geometric stress tensor

$$T^{(\rm geom)}_{\mu\nu} \equiv -\frac{c^4}{8\pi G}\mathcal{M}_{\mu\nu}. \tag{17}$$

For a homogeneous effective component,

$$p_{\rm geom} = w_{\rm geom}\rho_{\rm geom}c^2, \tag{18}$$

and accelerated FLRW expansion requires

$$\rho_{\rm geom} + \frac{3p_{\rm geom}}{c^2} < 0 \qquad \Longleftrightarrow \qquad w_{\rm geom} < -\frac{1}{3}. \tag{19}$$

This is an important constraint on DMET: ordinary attractive gravity from neighbouring masses is not, by itself, a dark-energy mechanism. The larger geometry must generate an effective curvature/stress contribution satisfying Eq. (19), or an inhomogeneous model must reproduce the observed distance–redshift relation without an FLRW dark-energy component.

A useful candidate mechanism is a dynamical strain field q with

$$\rho_q = \frac{1}{2}\dot{q}^2 + V(q), \qquad p_q = \frac{1}{2}\dot{q}^2 - V(q), \tag{20}$$

so potential-dominated strain has $p_q \simeq -\rho_q c^2$. This is a mathematically consistent way to translate the membrane-tension analogy into an effective negative-pressure sector, but it is an added model ingredient until derived from the bulk geometry.

7 Lensing must use the same geometry

A weak relativistic metric can be written

$$ds^2 = -\left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 + \left(1 - \frac{2\Psi}{c^2}\right) d\boldsymbol{x}^2. \tag{21}$$

Slow matter responds mainly to Φ, while light responds to $\Phi + \Psi$. The deflection angle is

$$\boldsymbol{\alpha} = \frac{1}{c^2} \int \nabla_\perp (\Phi + \Psi)\, d\ell. \tag{22}$$

DMET therefore cannot fit rotation curves with one potential and then choose a separate lensing halo. The same geometric completion must reproduce both dynamics and lensing.

8 The JWST / cross-region hypothesis is separate

The most speculative extension is that some apparently very early mature galaxies are actually galaxies in another universe-region whose photons reach us through a boundary or shared bulk geometry. Gravitational lensing and cross-region propagation must be distinguished.

A conventional lens changes apparent angle and flux through the lens mapping. Its magnification is

$$\mu_{\rm lens} = \frac{1}{|\det \mathsf{A}|}, \qquad \mathsf{A}_{ij} = \frac{\partial \beta_i}{\partial \theta_j}. \tag{23}$$

Lensing can make a source appear brighter and larger in angle, but a static gravitational lens does not create an arbitrary cosmological redshift. A genuine cross-region source therefore needs an explicit redshift/transfer law. In geometric optics,

$$\omega = -u_\mu k^\mu, \tag{24}$$

and any boundary model must calculate the ratio of emitted to observed frequency from the complete photon path. For a non-dispersive mechanism, every spectral feature must yield the same redshift.

Status. The statement "a large JWST galaxy may be a lensed galaxy from another universe-region" should therefore be treated as a testable extension, not as evidence for DMET. It survives only if one explicit boundary geometry predicts the observed redshift, magnification, image morphology, time dilation and sky distribution.

9 Minimum falsifiable programme

1. **Galaxy dynamics:** fit Eq. (2) to baryonic mass maps using one universal a_M.
2. **Rotation–lensing identity:** use the same relativistic potentials to predict lensing without an independent halo.
3. **Clusters:** test whether the geometry can reproduce merging-cluster lensing offsets without system-specific arbitrary memory terms.
4. **Early universe:** demonstrate a stable pressureless geometric mode capable of supporting CMB potentials and structure growth.
5. **Expansion:** derive $\mathcal{M}_{\mu\nu}$ or its equivalent from the larger geometry and reproduce $H(z)$ while satisfying isotropy constraints.
6. **Boundary optics:** only after the gravitational sector survives, calculate cross-region photon transfer and compare with JWST spectroscopy and lensing.

10 Bottom line

The revised mathematical statement of DMET is not “dark matter is a dent” and “dark energy is a pull from other universes.” It is more precise:

> Mass–energy creates curvature. DMET asks whether a larger connected geometry modifies the weak-field response around galaxies and supplies additional tidal/strain curvature on cosmological scales. Those effects must be derived from one covariant geometry and must reproduce motion, lensing, structure growth and expansion with a shared parameter set.

References

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